CHAPTER 37 Interference

Answers to Understanding the Concepts Questions

1. As discussed in the book, a single source of incoherent light with two small slits yields two sources of coherent light, but at the cost of a great reduction in intensity. A laser does not suffer from this disadvantage.

2. Constructive interferences occur wherever the two coherent waves add up to produce the greatest amplitude in the resultant wave. The only difference between a crest and a trough is that there is a negative sign in front of the electric field at the location of the trough. This certainly does not affect the amplitude of the wave, which is still a maximum since the two component waves are in phase. So yes, the troughs are also locations of constructive interference. As for the regions between the crests and troughs, since the net electric field is zero, we certainly have destructive interferences there.

3. The difference between a direct observation with the eye and using a screen to project the interference pattern is that the eye is a converging lens that concentrates the interference fringes. Even though in principle the fringes should be visible through the eye directly, they are often bunched so close together in the field of view that it is hard to tell them apart. It is therefore not practical to see the fringes directly without using a screen. There is, however, an important exception. With an optical grating, which has many thousands of slits per inch, the bright fringes are so sharp (well-defined) and so widely apart from each other, that it is entirely visible through the eye without a screen. In fact this is how an optical spectrometer works, without any screen.

4. Since the holes have circular symmetry, we would expect the resulting interference pattern to exhibit the same symmetry. So instead of lines we would expect concentric circular rings of alternating intensity (i.e., bright/dark circular rings).

5. Antireflecting coatings must have a thickness that is a quarter wavelength of the light transmitted, or an integer fraction of that. For attainable thicknesses it turns out that only one wavelength in the visible range of light will have a wavelength for which there is the required destructive interference.

6. The light from stars and planets must travel through the atmosphere to reach our eyes. Due to the inhomogeneities of the air, the light rays from a star move about on the retina, and the intensity varies, too, leading to the twinkling phenomenon. Since the planets are much closer to us than the stars, they subtend a much larger angle, so the variations in light intensity from different parts of the image tend to average out, and the plant appear steady. In outer space devoid of an atmosphere, there is no corresponding distortion of light, so the images from both stars and planets are steady.

7. Thin-film interferences can be observed whether or not from afar. The reason why we assume that the observation is made from afar is that it simplifies the mathematical analysis — when viewed from afar, light rays reflected from the film can be considered as essentially parallel to each other.

8. To observe a steady interference pattern, the participating light rays must be coherent. For natural sunlight, the coherence length is no more than several wavelengths, meaning that the path-length difference between two visible light rays usually cannot be more than a few microns for they to remain

coherent. For a thick film in which the path-length difference between different reflected light rays is many wavelengths, coherence is lost and no interference can be observed.

9. The energy density at a given point is proportional to $|\vec{E}(\vec{r},t)|^2$, and if the field vanishes there, so does the energy density. Energy conservation merely demands that the *total* energy — that is, $|\vec{E}(\vec{r},t)|^2$ integrated over the entire space — be conserved.

10. As a beam of light is incident upon a reflective surface, it is partially reflected and partially transmitted. The sum of the intensities of the reflected and transmitted beams equals that of the incident beam, as required by the conservation of energy. Thus if there is a minimum in reflected intensity at a certain wavelength, there must be a maximum in transmitted energy at the same wavelength, and vice versa. In fact, this is the reason why camera lenses are coated with an antireflective coating — minimizing reflection necessarily results in a maximization of the intensity of transmitted light.

11. The sum of two fields that takes the form $E_1\hat{i} + E_2\hat{j}$ cannot form an interference pattern. The square of the magnitude of this quantity is $E_1^2 + E_2^2$, whose time-average value is uniform. A cross term, in the form of E_1E_2, is essential for the existence of interference phenomena.

12. As m increases, the path-difference between the reflected light rays also increases. To observe interference fringes the light rays must remain coherent, which means the path-difference must be less than the coherence length of the light source. So the answer to this question depends on the quality of the light being used. A highly coherent source, such as that from a laser, can certainly allow us to observe all the fringes in this case. In any case the color will remain blue, since that only depends on the frequency of the source.

13. According to the solution to Problem 52, minimum reflection occurs when the index of refraction of the coating satisfies $n_{\text{air}}/n_{\text{coat}} = n_{\text{coat}}/n_{\text{glass}}$, or $n_{\text{coat}} = (n_{\text{air}}n_{\text{glass}})^{1/2}$. Obviously, this implies that n_{coat} is in between n_{air} and n_{glass}.

14. There is some reflection of light from the top of the curved piece of glass, and in principle there is interference between the light reflected at the top surface and that reflected at the curved surface, for example. However the path difference in the lens (in contrast to that in the narrow air gap) is many thousands of wavelengths. Since incoming light is never perfectly monochromatic, and since one would not necessarily go to the trouble of machining the top surface with the same precision as the curved surface, such interference patterns arc normally washed out.

15. The waves from both antennas interfere destructively where you stand, which is why the signal is weakened after the addition of the second antenna. There is no violation of energy conservation, though, since the intensity over other regions in space, where constructive interferences occur, the intensity is greatly enhanced (four times as much as that from a single antenna). Remember, conservation of energy only requires that the total power, that is, intensity integrated over the entire space, be the same as the sum of the powers produced by both antennas.

16. The condition for maximum interference is $d\sin\theta = m\lambda$, where $\sin\theta = y/(y^2+R^2)^{1/2}$. Thus $dy/(y^2+R^2)^{1/2} = m\lambda$, or $y = m\lambda(y^2+R^2)^{1/2}/d$. The spacing between the mth and (m +1)-th adjacent maxima is then $\Delta y = \Delta\,[m\lambda(y^2+R^2)^{1/2}/d] \approx \lambda(y^2+R^2)^{1/2}/d$, which is not uniform (as y increases with m).

17. The positions of maxima of interference patterns can generally be measured to very high precision. Therefore a tiny change in wavelength can be translated into a measurable change in the positions of the maxima.

18. The lenses are coated with an antireflective coating, designed to minimize the reflection of light whose wavelength is in the midrange of the visible light spectrum (yellow-green). The thickness of the film is such that it causes destructive interference for reflected light of this wavelength. The same thickness does not cause destructive interference in other wavelengths far from the midrange of the visible light spectrum, such as purple, which has nearly constructive interference for the same film thickness. Hence the purplish tint of the reflection.

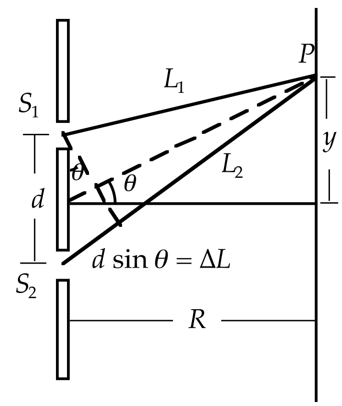
19. As indicated in the text, total destructive interference between light waves requires not only the proper phase relationship but also (nearly) equal amplitudes. The condition for this is worked out in Problem 52.

20. If each slit is very narrow, meaning that the slit width is considerably less than the wavelength, then one does not have to consider any phase difference of light from different parts of the same slit. If that's not the case and the width of the slit is comparable with the wavelength, then we must imagine that each slit is actually composed of a series of parallel, narrow ones, each of which acts as a source of light with different phase angles. Once then needs to sum up the contribution from all these narrow slits to find the resultant wave from that (wide) slit. This is beyond the phenomenon of interference and is referred to as diffraction, which we discuss in the next chapter. If the slit width is even wider, many times the wavelength, then one might lose coherence between the light from one edge of the slit and that from the other edge, in which case interference patterns are no longer present.

21. In order to exhibit interference patterns, light from various parts of the source must be coherent. For natural sunlight this means that the path-length of the light ray from one edge of the slit cannot differ from that from the other edge by more than a few wavelengths, or several microns. With mini-blinds that path-difference is of the order of centimeters, far beyond a few microns. The coherence condition is therefore not satisfied, and no interference is observed.

22. Actually lenses used in optical instruments are coated on both sides; the front coating is used to eliminate reflection at the front surface, and the back coating is used to eliminate the secondary reflection from the back side of the lens. The coating on the back surface will not eliminate reflection from the front surface for the reasons that appear in the answer to Question 14; similarly the coating on the front will not work to eliminate reflection from the back side.

Solutions to Problems



1. For constructive interference, the path-length difference is a multiple of the wavelength:
   $\Delta L = d \sin\theta = n\lambda, \quad n = 0, 1, 2, 3, \dots$.
   We find the location on the screen from
   $y = R \tan\theta$.
   For small angles, we have
   $\sin\theta \simeq \tan\theta$, which gives
   $y = R(n\lambda/d) = nR\lambda/d$.
   The separation of bright spots is
   $\Delta y = \Delta n\, R\lambda/d$;
   $0.70 \times 10^{-2}\text{ m} = (1)(3\text{ m})\lambda/(0.20 \times 10^{-3}\text{ m})$,
   which gives $\lambda = 4.7 \times 10^{-7}\text{ m} = \boxed{4.7 \times 10^2\text{ nm}}$.

2. For destructive interference, the path-length difference is an odd multiple of half the wavelength:
   $\Delta L = d \sin\theta = (2n - 1)\frac{1}{2}\lambda, \quad n = 1, 2, 3, \dots$.
   We find the location on the screen from
   $y = R \tan\theta$.
   For small angles, we have
   $\sin\theta \simeq \tan\theta$, which gives
   $y = R[(2n - 1)\lambda/2d] = (n - \frac{1}{2})R\lambda/d$.
   For the first minimum we have
   $y_1 = \frac{1}{2}(2.0\text{ m})(630 \times 10^{-9}\text{ m})/(0.25 \times 10^{-3}\text{ m}) = 5.4 \times 10^{-3}\text{ m} = \boxed{2.52\text{ mm}}$.

3. The wavelength of the water waves is $\lambda = v/f$. Because the wavelength is comparable to the slit separation, we cannot assume small angles. For constructive interference, the path-length difference is a multiple of the wavelength:
   $\Delta L = d \sin\theta = n\lambda = nv/f$.
   For the first maximum, $n = 1$, we have
   $(3.5 \times 10^{-2}\text{ m}) \sin\theta = (1)\,(0.12\text{ m/s})/(12\text{ Hz})$, which gives $\theta = 16.6°$.
   We find the location on the screen from
   $y = R \tan\theta = (0.8\text{ m}) \tan 16.6° = \boxed{0.24\text{ m}}$.

4. For constructive interference, the path-length difference is a multiple of the wavelength:
   $\Delta L = d \sin\theta = n\lambda$.
   We find the location on the screen from $y = R \tan\theta$.
   For small angles, we have
   $\sin\theta \simeq \tan\theta$, which gives
   $y = R(n\lambda/d) = nR\lambda/d$.
   For the third maximum, $n = 3$, we have
   $y = (3)(40 \times 10^{-2}\text{ m})(525 \times 10^{-9}\text{ m})/(120 \times 10^{-6}\text{ m}) = 5.3 \times 10^{-3}\text{ m} = \boxed{5.3\text{ mm}}$.

5. For constructive interference, the path-length difference is a multiple of the wavelength:
   $\Delta L = d \sin\theta = n\lambda$.
   We find the location on the screen from $y = R \tan\theta$. For small angles, we have
   $\sin\theta \simeq \tan\theta$, which gives
   $y = R(n\lambda/d) = nR\lambda/d$.
   For the third maximum, $n = 3$, we have
   $18 \times 10^{-2}\text{ m} = (3)R(590 \times 10^{-9}\text{ m})/(0.12 \times 10^{-3}\text{ m})$, which gives $R = \boxed{12.2\text{ m}}$.

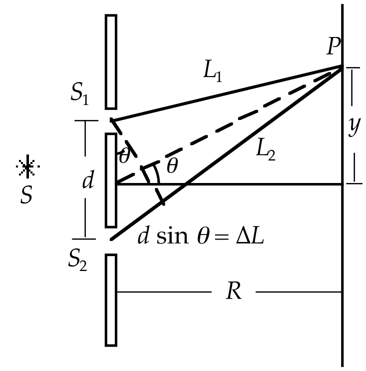
6. Because there is no path difference on the line perpendicular to the line joining the speakers, the intensity will be a $\boxed{\text{maximum}}$ there.
   The wavelength of the sound is $\lambda = v/f = (330\text{ m/s})/(380\text{ Hz}) = 0.868\text{ m}$. If the microphone is moved to the next maximum, the path difference must be one wavelength:
   $\Delta L = d \sin \theta = \lambda;$
   $(65 \times 10^{-2}\text{ m}) \sin \theta = 0.868\text{ m}$, Maximum s occur at $\theta = +90^0, -90^0$.
   The arc length is
   $s = R\pi = (2.5\text{ m})(3.14) = \boxed{7.9\text{ m}}$.

$\boxed{7}$. For constructive interference, the path-length difference is a multiple of the wavelength:
   $\Delta L = d \sin \theta = n\lambda.$
   We find the location on the screen from $y = R \tan \theta$.
   For small angles, we have
   $\sin \theta \simeq \tan \theta$, which gives $y = R(n\lambda/d) = nR\lambda/d$.
   The separation of bright spots is
   $\Delta y = \Delta n\, R\lambda/d.$
   (a) When $\lambda_2 = 2\lambda_1$, we have $\Delta y_2 = 2\,\Delta y_1$; the separation $\boxed{\text{doubles}}$.
   (b) When $d_2 = 2d_1$, we have $\Delta y_2 = \frac{1}{2}\,\Delta y_1$; the separation $\boxed{\text{reduces by } \tfrac{1}{2}}$.
   (c) When $R_2 = 2R_1$, we have $\Delta y_2 = 2\,\Delta y_1$; the separation $\boxed{\text{doubles}}$.
   (d) The separation of maxima does not depend on the intensity; there is $\boxed{\text{no change}}$.

8. For constructive interference, the path-length difference is a multiple of the wavelength:
   $\Delta L = d \sin \theta = n\lambda.$
   We find the location on the screen from $y = R \tan \theta$.
   For small angles, we have
   $\sin \theta \simeq \tan \theta$, which gives $y = R(n\lambda/d) = nR\lambda/d$.
   The separation of bright maxima is
   $\Delta y = \Delta n\, R\lambda/d;$
   $0.50 \times 10^{-2}\text{ m} = (1)R(595 \times 10^{-9}\text{ m})/(0.15 \times 10^{-3}\text{ m})$, which gives $R = \boxed{1.3\text{ m}}$.

9. For constructive interference, the path-length difference is a multiple of the wavelength:
   $\Delta L = d \sin \theta = n\lambda.$
   We find the location on the screen from $y = R \tan \theta$.
   For small angles, we have
   $\sin \theta \simeq \tan \theta$, which gives $y = R(n\lambda/d) = nR\lambda/d$.
   The separation of bright maxima is
   $\Delta y = \Delta n\, R\lambda/d.$
   With 8 maxima on either side of the central maximum, there are 16 separations, so we have
   $16.8 \times 10^{-2}\text{ m} = (16)(3.45\text{ m})\lambda/(0.21 \times 10^{-3}\text{ m})$, which gives
   $\lambda = 6.4 \times 10^{-7}\text{ m} = \boxed{6.4 \times 10^2\text{ nm}}$.

10. The phase difference at point P is due to the additional path length from the source to the slits and the additional distance to the screen:



$\Delta L = \frac{1}{2}\lambda + d \sin\theta$.

For maxima, this path-length difference must be a multiple of λ:

$\frac{1}{2}\lambda + d \sin\theta_{\max} = n\lambda$, or

$d \sin\theta_{\max} = (n - \frac{1}{2})\lambda$, $n = 0, \pm 1, \pm 2, \ldots$.

The locations of the maxima are

$y_{\max} = R \tan\theta_{\max}$.

For small angles, we have

$\sin\theta \simeq \tan\theta$, which gives

$y_{\max} = \boxed{(n - \frac{1}{2})R\lambda/d, \quad n = 0, \pm 1, \pm 2, \ldots}$.

For minima, this path-length difference must be an odd multiple of a half-wavelength:

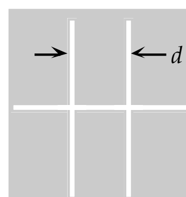
$\frac{1}{2}\lambda + d \sin\theta_{\min} = (n + \frac{1}{2})\lambda$, or $d \sin\theta_{\min} = n\lambda$, $n = 0, \pm 1, \pm 2, \ldots$.

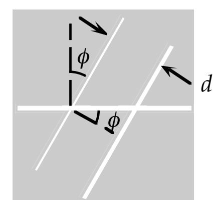
The locations of the minima are

$y_{\min} = R \tan\theta_{\min} = \boxed{nR\lambda/d, \quad n = 0, \pm 1, \pm 2, \ldots}$.

We see that the point at $\theta = 0$ will have $n = 0$ for destructive interference, so it will be a $\boxed{\text{minimum}}$.

11. The overlapping slits form a system of two point sources. As the double slit is rotated, the distance between the sources changes. From the figure we have





$d' = d/\cos\phi$.

If we use the result from Problem 10 without the initial $\frac{1}{2}\lambda$ difference, we have

$y_{\max} = nR\lambda/d' = nR\lambda(\cos\phi)/d$, $n = 0, \pm 1, \pm 2, \ldots$.

As ϕ increases, $\cos\phi$ and thus $y_{\max}$ decreases; the fringes move $\boxed{\text{inward}}$.

12. The wavelength of the waves is $\lambda = c/f$. The phase difference at point P is due to the phase difference between the sources and the additional distance to the screen. Because each wavelength is equivalent to a phase difference of 2π, for maxima we have

$\phi = \alpha + [2\pi(d \sin\theta)/\lambda] = n2\pi$, $n = 0, \pm 1, \pm 2, \ldots$.

The location of the first maximum is

$y = R \tan\theta \simeq R \sin\theta = (R\lambda/d)(1 - \alpha/2\pi) = (Rc/fd)(1 - \alpha/2\pi)$

$= [(1.6 \text{ m})(3 \times 10^8 \text{ m/s})/(3.7 \times 10^{10} \text{ Hz})(18\times 10^{-2} \text{ m})](1 - \alpha/2\pi)$

$= (0.072 \text{ m})(1 - \alpha/2\pi) = \boxed{7.2\,(1 - \alpha/2\pi) \text{ cm}}$.

When $\alpha = 0$, $y = 7.2$ cm; and when $\alpha = 2\pi$, $y = 0$ cm.

13. If we use the result from Problem 12, with v for the velocity, the position of the first maximum is

$y = (Rv/fd)(1 - \alpha/2\pi)$.

The difference in frequency will create a phase difference which is a function of time:

$\alpha = (\Delta\omega)t = 2\pi t\,\Delta f$, so we have $y = (Rv/fd)(1 - t\,\Delta f)$.

We find the speed of the fringe movement from

$v_{\text{fringe}} = |dy/dt| = (Rv/d)\,\Delta f/f$.

For the ripple tank we have

$v_{\text{fringe,ripple}} = [(1 \text{ m})(0.15 \text{ m/s})/(0.05 \text{ m})](10^{-6}) = \boxed{3 \times 10^{-6} \text{ m/s}}$.

For the optical experiment we have

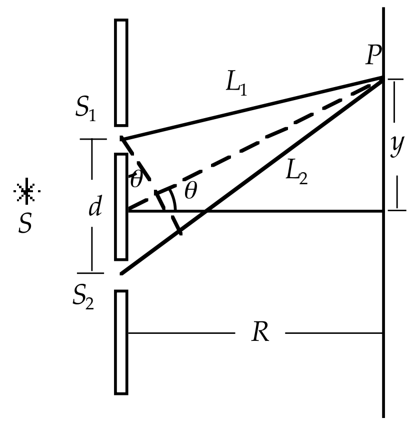
$v_{\text{fringe,optical}} = [(1 \text{ m})(3.0 \times 10^8 \text{ m/s})/(0.25\ 10^{-3} \text{ m})](10^{-6}) = \boxed{1.2 \times 10^6 \text{ m/s}}$.

Because the speed for the ripple tank is unnoticeable, the frequency difference can be much larger, which means less coherence, before the effect is noticeable. In the optical experiment, the fringe pattern would be completely blurred. The coherence of the two sources must be many orders of magnitude better to have a noticeable interference pattern.

14. For small angles, the locations of the maxima of λ_1 are given by
    $y_{\max} = n\lambda_1 R/d, \quad n = 0, \pm 1, \pm 2, \ldots.$
    The locations of the minima of λ_2 are given by
    $y_{\min} = (n + \frac{1}{2})\lambda_2 R/d, \quad n = 0, \pm 1, \pm 2, \ldots .$
    We are given that
    $y_{\max\ 20} = y_{\min\ 19}$;
    $20\lambda_1 R/d = (19 + \frac{1}{2})\lambda_2 R/d$, which gives $\lambda_2 = 1.0256\lambda_1$.
    The relative difference is small:
    $(\lambda_1 - \lambda_2)/\lambda_1 = (1 - 1.0256)\lambda_1/\lambda_1 = \boxed{-0.0256}$.

15. (a) When $\theta = 0$, we find the paths from the two slits to the screen from
    $L_1{}^2 = R^2 + (d/2)^2$ and $L_2{}^2 = R^2 + (d/2)^2$.
    Because $L_1 = L_2$, we have $\Delta L = 0$, constructive interference, and thus a maximum.

    (b) The path-length difference at the point P is
    $$\Delta L = \sqrt{\left[y + (d/2)\right]^2 + R^2} - \sqrt{\left[y - (d/2)\right]^2 + R^2}.$$
    For the first maximum, this path-length difference must be a wavelength. If we substitute $y = R\tan\theta$, we have
    $$\lambda = \sqrt{\left[R\tan\theta + (d/2)\right]^2 + R^2} - \sqrt{\left[R\tan\theta - (d/2)\right]^2 + R^2}, \text{ or}$$
    $$\sqrt{\left[R\tan\theta + (d/2)\right]^2 + R^2} = \sqrt{\left[R\tan\theta - (d/2)\right]^2 + R^2} + \lambda.$$
    When we square both sides and cancel common terms, we have
    $$2\lambda\sqrt{\left[R\tan\theta - (d/2)\right]^2 + R^2} = 2Rd\tan\theta - \lambda.$$
    When we square again and cancel common terms, we get
    $$4\left(d^2 - \lambda^2\right)R^2\tan^2\theta = \lambda^2 d^2 + 4\lambda^2 R^2 - \lambda^4, \text{ which gives}$$
    $$\tan^2\theta = \lambda^2\left(\frac{1}{d^2 - \lambda^2} + \frac{1}{4R^2}\right), \text{ so the angle for the first maximum is}$$
    $$\theta = \tan^{-1}\left(\lambda\sqrt{\frac{1}{d^2 - \lambda^2} + \frac{1}{4R^2}}\right).$$



    (c) When $R \gg d$ and $d \gg \lambda$, we have
    $\tan\theta \simeq \lambda/d \ll 1$.
    For small angles, we have
    $\tan\theta \simeq \sin\theta \simeq \lambda/d$, which is the distant-screen result.

16. We find the angle from the central line, which will be small, from
    $y = R\tan\theta \simeq R\sin\theta$.
    The phase difference on the screen is
    $\phi = 2\pi(\Delta L/\lambda) = 2\pi(d\sin\theta)/\lambda \simeq 2\pi yd/\lambda R$.
    At a distance 0.6 mm from the central maximum, we have
    $\phi_1 = 2\pi(0.6 \times 10^{-3}\ \text{m})(0.3 \times 10^{-3}\ \text{m})/(500 \times 10^{-9}\ \text{m})(1\ \text{m}) = 2.3$ rad.
    The intensity is
    $I_1 = 4I_0\cos^2(\frac{1}{2}\phi_1) = 4I_0\cos^2[\frac{1}{2}(2.3\ \text{rad})]$, which gives $I_1/I_0 = \boxed{0.73}$.
    At a distance – 0.5 mm from the central maximum, we have
    $\phi_2 = 2\pi(-0.5 \times 10^{-3}\ \text{m})(0.3 \times 10^{-3}\ \text{m})/(500 \times 10^{-9}\ \text{m})(1\ \text{m}) = -1.88$ rad.
    The intensity is
    $I_2 = 4I_0\cos^2(\frac{1}{2}\phi_1) = 4I_0\cos^2[\frac{1}{2}(-1.88\ \text{rad})]$, which gives $I_2/I_0 = \boxed{1.4}$.

17. The intensity is

$$I = 4I_0 \cos^2(\tfrac{1}{2}\phi) = 4(1.0 \times 10^3\ \mathrm{W/m^2}) \cos^2(\tfrac{1}{2}60°) = \boxed{3.0 \times 10^3\ \mathrm{W/m^2}}.$$

18. We find the phase difference from

$$I = 4I_0 \cos^2(\tfrac{1}{2}\phi);$$

$$2I_0 = 4I_0 \cos^2(\tfrac{1}{2}\phi),\ \text{which gives}\ \phi = \boxed{90°}.$$

$\boxed{19}$. We find the phase difference from

$$I = 4I_0 \cos^2(\tfrac{1}{2}\phi);$$

$$\tfrac{1}{4}I_0 = 4I_0 \cos^2(\tfrac{1}{2}\phi),\ \text{which gives}\ \phi = \boxed{151°}.$$

For the double slit, we have

$$\phi = (2\pi d/\lambda) \sin\theta;$$

$$(151°)[(\pi\ \mathrm{rad})/(180°)] = [2\pi(485\ \mathrm{nm})/\lambda] \sin 27°,\ \text{which gives}\ \lambda = \boxed{525\ \mathrm{nm}}.$$

20. From the intensity for the double slit,

$$I = 4I_0 \cos^2[(\pi d \sin\theta)/\lambda]$$

the first maximum is given by

$$(\pi d \sin\theta)/\lambda \simeq \pi d\theta/\lambda = \pi,\quad \text{or}$$

$$d = \lambda/\theta = (490\ \mathrm{nm})/(1.7 \times 10^{-3}\ \mathrm{rad}) = \boxed{0.29\ \mathrm{mm}}.$$

21. To distinguish the slit separation from differentials, we let the separation be D. When the screen is far from the slits, the angles are small, so the intensity a distance y from the central maximum becomes

$$I = 4I_0 \cos^2[(\pi D \sin\theta)/\lambda] \simeq 4I_0 \cos^2(\pi y D/\lambda R).$$

The average intensity over the screen is

$$I_{av} = \frac{\int_{\text{screen}} I\,dy}{\int_{\text{screen}} dy} = \frac{\int_{\text{screen}} 4I_0 \cos^2(\pi y D/\lambda R)\,dy}{\int_{\text{screen}} dy}.$$

If we change variable to $\beta = \pi y D/\lambda R$, we have $d\beta = (\pi D/\lambda R)\,dy$, and the average intensity becomes

$$I_{av} = \frac{\int_{\text{screen}} 4I_0(\lambda R/\pi D) \cos^2\beta\,d\beta}{\int_{\text{screen}} (\lambda R/\pi D)\,d\beta} = 4I_0 \langle\cos^2\beta\rangle.$$

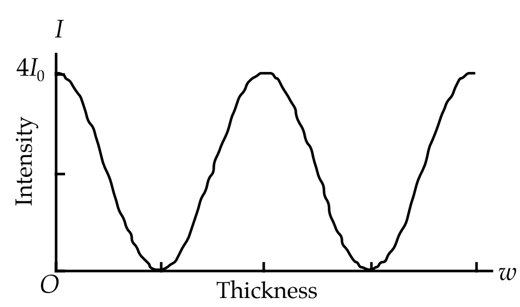
For many fringes on the screen, β varies over many cycles, and the average value of $\cos^2\beta$ is $\tfrac{1}{2}$.
The average intensity is

$$I_{av} = 4I_0 \langle\cos^2\beta\rangle = 4I_0(\tfrac{1}{2}) = \boxed{2I_0}.$$

22. The wavelength of the light in the liquid is $\lambda = \lambda_0/n$. Because the physical distance to the point C is the same for the two slits, the phase difference is produced by the difference in the number of wavelengths in the distance w with and without the liquid:

$$\phi = 2\pi(w/\lambda - w/\lambda_0) = 2\pi(n-1)w/\lambda_0.$$

The intensity at the point C is
$I = 4I_0 \cos^2(\phi/2) = 4I_0 \cos^2[(n-1)w\pi/\lambda_0]$.
As expected, when $w = 0$, the intensity is a maximum.
Maxima occur when $w = [\lambda_0/(n-1)]m,\quad m = 0, 1, 2, \ldots$.
Minima occur when $w = [\lambda_0/2(n-1)]m,\quad m = 1, 3, 5, \ldots$.



23. The intensity of the pattern is

$$I = 4I_0 \cos^2 [(\pi d \sin \theta)/\lambda].$$

At the central maximum, we have $\theta = 0$ and $I = 4I_0$.

We find the angle where the intensity is half its maximum value from

$$I = \tfrac{1}{2}(4I_0) = 4I_0 \cos^2 [(\pi d \sin \theta_{1/2})/\lambda], \text{ or } \cos [(\pi d \sin \theta_{1/2})/\lambda] = 1/\sqrt{2};$$

$(\pi d \sin \theta_{1/2})/\lambda = \pi/4$, which gives $\theta_{1/2} = \sin^{-1} (\lambda/4d)$.

The angular width is twice this angle:

$$\Delta\theta = \boxed{2 \sin^{-1} (\lambda/4d)}.$$

The other angles where the intensity is $2I_0$ are given by

$(\pi d \sin \theta)/\lambda = 3\pi/4, 5\pi/4, \ldots$.

In general, the widths will not be the same, except near the central maximum, where $\sin \theta \simeq \theta << 1$.

24. We use the result from Problem 23:

$\sin \theta = \lambda/4d = (633 \times 10^{-9} \text{ m})/[4(0.35 \times 10^{-3} \text{ m})]$, which gives $\theta = \boxed{0.026°}$.

The full width at half-maximum is

$$\Delta y = 2R \tan \theta = 2(1.8 \text{ m}) \tan 0.026° = 1.6 \times 10^{-3} \text{ m} = \boxed{1.6 \text{ mm}}.$$

$\boxed{25}$. (a) Because the two sources are in phase, the maximum will occur where the path-length difference is zero, which is $\boxed{\text{along the perpendicular bisector}}$.

(b) The maximum intensity is

$$I_{max} = 4I_0 = 4(5.0 \times 10^{-4} \text{ W/m}^2) = \boxed{2.0 \times 10^{-3} \text{ W/m}^2}.$$

(c) We find the angle where the intensity is half its maximum value from

$$I = \tfrac{1}{2}(4I_0) = 4I_0 \cos^2 [(\pi d \sin \theta_{1/2})/\lambda], \text{ or } \cos [(\pi d \sin \theta_{1/2})/\lambda] = 1/\sqrt{2};$$

$\sin \theta_{1/2} = \lambda/4d = c/4fd = (3.0 \times 10^8 \text{ m/s})/[4(3.8 \times 10^7 \text{ Hz})(12 \text{ m})] = 0.16$, which gives $\theta_{1/2} = \boxed{9.5°}$.

26. The resultant electric field at the screen is

$$E_{net} = E_1 \sin(\omega t) + E_2 \sin(\omega t + \phi).$$

The net intensity is the time average of the magnitude of the Poynting vector:

$$\begin{aligned}\langle I_{net}\rangle &= \langle S\rangle = c\varepsilon_0\langle[E_1 \sin(\omega t) + E_2 \sin(\omega t + \phi)]^2\rangle \\ &= c\varepsilon_0\langle[E_1 \sin(\omega t)]^2\rangle + c\varepsilon_0\langle[E_2 \sin(\omega t + \phi)]^2\rangle + c\varepsilon_0\langle 2E_1E_2 \sin(\omega t) \sin(\omega t + \phi)\rangle.\end{aligned}$$

We use $\langle\sin^2 \theta\rangle = \tfrac{1}{2}$ and expand $\sin(\omega t + \phi)$:

$$\begin{aligned}\langle I_{net}\rangle &= \tfrac{1}{2}c\varepsilon_0 E_1{}^2 + \tfrac{1}{2}c\varepsilon_0 E_2{}^2 + 2c\varepsilon_0 E_1E_2\langle\sin(\omega t)[\sin(\omega t) \cos \phi + \cos(\omega t) \sin \phi]\rangle \\ &= \tfrac{1}{2}c\varepsilon_0 E_1{}^2 + \tfrac{1}{2}c\varepsilon_0 E_2{}^2 + 2c\varepsilon_0 E_1E_2[\langle\sin^2(\omega t)\rangle \cos \phi + \langle\sin(\omega t) \cos(\omega t)\rangle \sin \phi]. \\ &= \tfrac{1}{2}c\varepsilon_0 E_1{}^2 + \tfrac{1}{2}c\varepsilon_0 E_2{}^2 + 2c\varepsilon_0 E_1E_2(\tfrac{1}{2}) \cos \phi + 0.\end{aligned}$$

For the intensity from the individual slits, we have

$\langle I_1\rangle = \tfrac{1}{2}c\varepsilon_0 E_1{}^2$ and $\langle I_2\rangle = \tfrac{1}{2}c\varepsilon_0 E_2{}^2$, so we get

$$\langle I_{net}\rangle = \langle I_1\rangle + \langle I_2\rangle + 2(\langle I_1\rangle\langle I_2\rangle)^{1/2} \cos \phi.$$

27. For distances much larger than $d = 2.5$ m, the amplitudes of the electric fields are about the same. The path-length difference from the two slits is

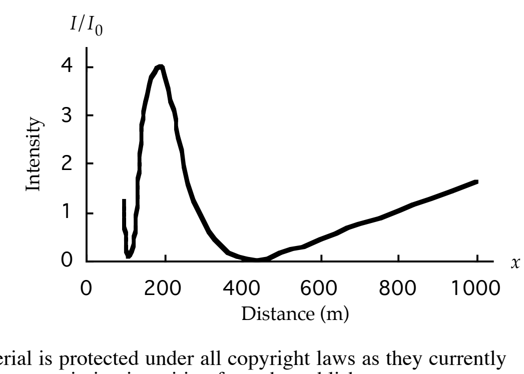
$$\begin{aligned}\Delta L &= (x^2 + d^2)^{1/2} - x = x[1 + (d^2/x^2)]^{1/2} - x \\ &\simeq x + (d^2/2x) - x = d^2/2x.\end{aligned}$$

Because the sources are out of phase by 45° or $\pi/4$ rad, the phase difference is

$$\phi = 2\pi\, \Delta L/\lambda + \pi/4 = \pi d^2/x\lambda + \pi/4.$$

The intensity is

$$\begin{aligned}I &= 4I_0 \cos^2 (\phi/2) = 4I_0 \cos^2 [(\pi d^2/2x\lambda) + \pi/8] \\ &= 4I_0 \cos^2 \{\pi(2.5 \text{ m})^2/[2x(0.020 \text{ m})] + \pi/8\} \\ &= 4I_0 \cos^2 (156\pi/x + \pi/8).\end{aligned}$$



28. If P_0 is the power output of each source, the intensities decrease as $1/r^2$:

$\langle I_1\rangle = P_0/4\pi r_1{}^2$ and $\langle I_2\rangle = P_0/4\pi r_2{}^2$, where $r_1 = x$, and $r_2 = (x^2 + d^2)^{1/2}$.

We use the analysis of Problem 27 for the phase difference:

$$\phi = \frac{2\pi}{\lambda}\left(\sqrt{d^2 + x^2} - x\right) + \frac{\pi}{4}.$$

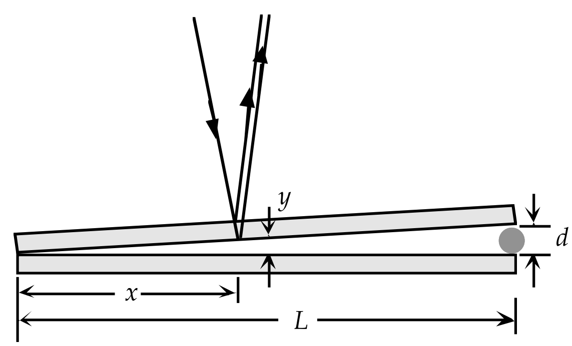
We use the result of Problem 26 for the net intensity:

$$\begin{aligned}\langle I_{\text{net}}\rangle &= \langle I_1\rangle + \langle I_2\rangle + 2\sqrt{\langle I_1\rangle\langle I_2\rangle}\cos\phi\\ &= \frac{P_0}{4\pi r_1^2} + \frac{P_0}{4\pi r_2^2} + 2\sqrt{\frac{P_0}{r_1^2}\frac{P_0}{r_2^2}}\cos\phi\\ &= \frac{P_0}{4\pi}\left\{\frac{1}{x^2} + \frac{1}{x^2 + d^2} + 2\sqrt{\frac{1}{x^2\left(x^2 + d^2\right)}}\cos\left[\frac{2\pi}{\lambda}\left(\sqrt{d^2 + x^2} - x\right) + \frac{\pi}{4}\right]\right\}\\ &= \frac{P_0}{4\pi x^2}\left\{1 + \frac{1}{1 + \left(d^2/x^2\right)} + \frac{2}{\sqrt{1 + \left(d^2/x^2\right)}}\cos\left[\frac{2\pi}{\lambda}\left(\sqrt{d^2 + x^2} - x\right) + \frac{\pi}{4}\right]\right\}.\end{aligned}$$

When $x >> d$, this reduces to

$$\langle I_{\text{net}}\rangle = \frac{2P_0}{4\pi x^2}\left[1 + \cos\left(\frac{\pi d^2}{x\lambda} + \frac{\pi}{4}\right)\right].$$

29. There is a phase difference for the reflected waves from the path-length difference, $(2y/\lambda)2\pi$, and the reflection at the bottom surface, π.



For constructive interference, we have

$\phi = (2y/\lambda)2\pi + \pi = m2\pi$, m = 1, 2, 3, … .

The thicknesses of the air wedge where the maxima occur are given by

$y = \frac{1}{4}\lambda(2m - 1)$, with $m = 1$ being the first maximum.

If a fringe is considered to be from a maximum to the next maximum, 102 fringes requires that $m = 103$ at the wire:

$d = \frac{1}{4}\lambda(2m - 1)$

$= \frac{1}{4}(656\text{ m})[2(103) - 1] = 3.36 \times 10^4\text{ nm} = \boxed{33.6\ \mu\text{m}}$.

30. There is a phase difference for the reflected waves from the path-length difference, $(2y/\lambda)2\pi$, and the reflection at the bottom surface, π. For constructive interference, we have

$\phi = (2y/\lambda)2\pi + \pi = m2\pi$, $m = 1, 2, 3, \ldots$.

The thicknesses of the air wedge where the maxima occur are given by

$y = \frac{1}{4}\lambda(2m - 1)$, with $m = 1$ being the first maximum.

For N fringes, the change in thickness is

$\Delta y = \frac{1}{4}\lambda(2\,\Delta m) = \frac{1}{2}\lambda N = \Delta x \tan\theta$, where θ is the angle of the wedge;

$\frac{1}{2}(615 \times 10^{-9}\text{ m})(17) = (1 \times 10^{-2}\text{ m})\tan\theta$, which gives $\theta = \boxed{0.030°}$.

$\boxed{31}$. For a bubble with thickness t, there is a phase difference for the reflected waves from the path-length difference, $(2t/\lambda)2\pi$, and the reflection at the top surface, π.

For constructive interference, we have

$\phi = (2t/\lambda_{\text{soap}})2\pi + \pi = m2\pi$, $m = 1, 2, 3, \ldots$, or

$t = (m - \frac{1}{2})\lambda/2n_{\text{soap}} = (m - \frac{1}{2})(420\text{ nm})/[2(1.3)] = \boxed{81\text{ nm}, 242\text{ nm}, 404\text{ nm}}$.

32. At each surface the wave is reflecting from a higher index, so there will be a π phase shift at each surface. For an oil film with thickness t, there is a phase difference for the reflected waves from the path-length difference, $(2t/\lambda_{\text{oil}})2\pi$. For constructive interference, the net phase change is

$$\phi = (2t/\lambda_{\text{oil}})2\pi + \pi - \pi = m2\pi,\ \ m = 1, 2, 3, \ldots,\ \ \text{or}$$

$$t = \boxed{m\lambda/2n_{\text{oil}}\ ,\ \ m = 1, 2, 3, \ldots}\ .$$

33. You want to maximize reflection, so the thickness t of the film should satisfy

$$2t = m\lambda_n = m\lambda n\ ;$$

$$t = \tfrac{1}{2}m\lambda_n = \tfrac{1}{2}m(\lambda/n) = \tfrac{1}{2}[(550\text{ nm})/(1.32)]m = (208\text{ nm})m, \text{ where } m = 1, 2, 3, \ldots\ .$$

The minimum thickness is therefore $\boxed{208\text{ nm}}$ (for $m = 1$). Two other thicknesses that also work are $\boxed{416\text{ nm}}$ (for $m = 2$) and $\boxed{625\text{ nm}}$ (for $m = 3$).

34. (a) The index of refraction of the oil coating is greater than both that of air (1.00) and water (1.33), so we expect a π-phase shift in the reflection from the top surface of the oil film, but not from the bottom surface of the oil film. The condition for destructive interference is then

$$2t = m\lambda_{\text{oil}} = m\lambda n\ ;$$

$$t = \tfrac{1}{2}m\lambda_{\text{oil}} = \tfrac{1}{2}m(\lambda/n) = \tfrac{1}{2}[(520\text{ nm})/(1.38)]m = (188\text{ nm})m, \text{ where } m = 1, 2, 3, \ldots\ .$$

The minimum thickness of the oil film is then $\boxed{188\text{ nm}}$ (for $m = 1$).

(b) The frequency corresponds to the wavelength $\lambda = 520$ nm (measured in air), so

$$f = c/\lambda = (3.00 \times 10^8\text{ m/s})/(520 \times 10^{-9}\text{ m}) = \boxed{5.77 \times 10^{14}\text{ Hz}}.$$

35. At a distance r from the center of the lens, the thickness of the air space is h, and the phase difference for the reflected waves from the path-length difference and the reflection at the bottom surface is

$$\phi = (2h/\lambda)2\pi - \pi.$$

For the first dark ring, we have

$$\phi = (2h/\lambda)2\pi - \pi = \pi, \quad \text{or}$$

$$h = \tfrac{1}{2}\lambda.$$

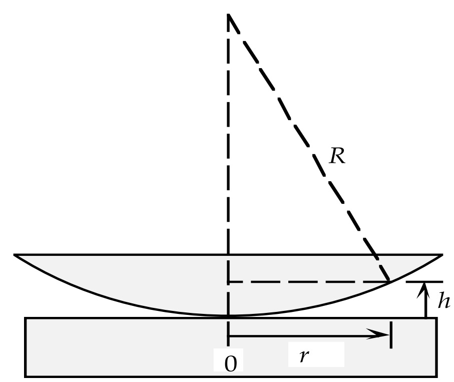
From the triangle in the diagram, we have

$$r^2 + (R - h)^2 = R^2, \ \text{ or } \ r^2 = 2hR - h^2 \simeq 2hR, \text{ when}$$

$h << R$.

We find the radius of the first dark ring from

$$r^2 = 2(\tfrac{1}{2}\lambda)R = \lambda R, \ \text{ which gives } \boxed{r = (\lambda R)^{1/2}}.$$



36. With respect to the incident wave, the wave that reflects from air at the top surface of the wedge has no phase change. With respect to the incident wave, the wave that reflects from glass at the bottom surface of the wedge has a phase change of π. Because the additional path through the wedge is negligible where the plates are in contact, the net phase difference between the two reflected waves is π, so the first band is dark.

37. The theory of this problem has been worked out in Example 37-5. With the data provided in this problem, the fringe separation is

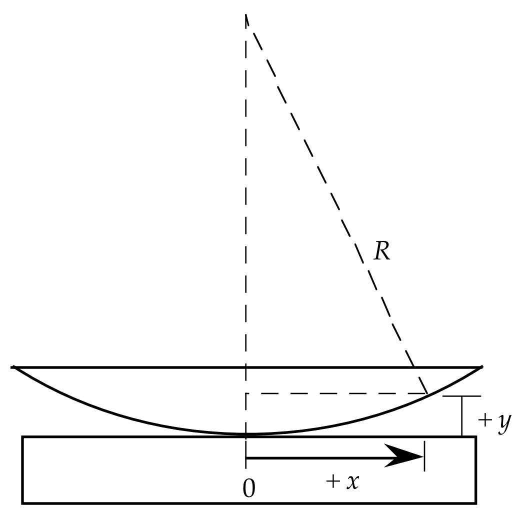
$$\Delta x = L\lambda/2d = (8.0\text{ cm})(0.655\ \mu\text{m})/[2(0.10\text{ cm})] = \boxed{26\ \mu\text{m}}.$$

38. We know that there is a dark fringe where the two glass plates touch each other. Away from that, with each additional dark fringe the separation between the two plates increases by $\lambda/2$. Since there are a total of $N = 109$ additional dark fringes, the separation between the two plates at the 110th dark fringe, near the other end, is
    $N(\lambda/2) = (109)[(0.590\ \mu\text{m})/2] = 32.2\ \mu\text{m}$.
    The diameter of the wire separating the two plates must exceed this value.
    Also, note that there are only 110 fringes, not 111, so the diameter of the wire cannot exceed
    $(N + 1)(\lambda/2) = (110)[(0.590\ \mu\text{m})/2] = 32.5\ \mu\text{m}$.
    Thus the diameter d of the wire must be in the range
    $\boxed{32.2\ \mu\text{m} < d < 32.5\ \mu\text{m}}$.

39. When the separation of the plate and the center of the lens is D, the phase difference is
    $\phi = (2D/\lambda)2\pi + \pi$.
    For the maxima, this must equal $m2\pi$. Because the spot was dark initially, m counts the number of maxima that pass:
    $(2D/\lambda)2\pi + \pi = m2\pi$, or
    $m = (2D/\lambda) + \tfrac{1}{2} = 2(0.25 \times 10^{-3}\ \text{m})/(500 \times 10^{-6}\ \text{m}) + \tfrac{1}{2} = (1.0 \times 10^{3}) + \tfrac{1}{2}$.
    Thus $\boxed{\text{1000 maxima pass}}$.
    A particular fringe corresponds to a particular phase difference. As the thickness of the air layer increases away from the center, the phase difference increases. When the lens is pulled away, the increase in thickness means a particular phase difference occurs closer to the center, so the corresponding fringe moves closer to the center. The rings $\boxed{\text{move in to the center}}$.

40. Because $n_{\text{water}} < n_{\text{glass}}$, the phase difference is the same as with air, except for the change in wavelength. With water in the space, the phase differences for the maxima are
    $\phi = (2y/\lambda_{\text{water}})2\pi + \pi = (2yn/\lambda)2\pi + \pi = m2\pi,\quad m = 1, 2, 3, \ldots$.
    A particular fringe corresponds to a particular phase difference. The increase in index means that the fringe will occur at a smaller y. The rings move in to the center.
    We find the change in the number of maxima at the center when the water is added from
    $\Delta\phi = (2nD/\lambda)2\pi + \pi - (2D/\lambda)2\pi + \pi = 2\pi\,\Delta m$;
    $\Delta m = (2D/\lambda)(n - 1) = [2(0.85 \times 10^{-3}\ \text{m})/(500 \times 10^{-6}\ \text{m})](1.33 - 1) = 1122$.
    Thus $\boxed{\text{1122 maxima pass at the center}}$.

41. At a distance x from the center of the lens, the thickness of the air space is y, and the phase difference for the reflected waves from the path-length difference and the reflection at the bottom surface is
    $\phi = (2y/\lambda)2\pi + \pi$.
    For the dark rings, we have
    $\phi = (2y/\lambda)2\pi + \pi = (m + \tfrac{1}{2})2\pi,\quad m = 0, 1, 2, 3, \ldots,$ or
    $y = \tfrac{1}{2}m\lambda,\quad m = 0, 1, 2, 3, \ldots,$
    where $m = 0$ is at the center of the lens.
    From the triangle in the diagram, we have
    $x^2 + (R - y)^2 = R^2$, or $x^2 = 2yR - y^2 \simeq 2yR$,
    when $y << R$.
    The diameter of the dark ring is $2x$, so we have
    $(\tfrac{1}{2}D)^2 = 2(\tfrac{1}{2}m\lambda)R = m\lambda R$.



    For the fourth dark ring, we have
    $(\tfrac{1}{2}D_4)^2 = (4)(520 \times 10^{-9}\ \text{m})(5.0\ \text{m})$, which gives $D_4 = 6.4 \times 10^{-3}\ \text{m} = \boxed{6.4\ \text{mm}}$.
    For the seventh dark ring, we have
    $(\tfrac{1}{2}D_7)^2 = (7)(520 \times 10^{-9}\ \text{m})(5.0\ \text{m})$, which gives $D_7 = 8.5 \times 10^{-3}\ \text{m} = \boxed{8.5\ \text{mm}}$.

42. At a distance x from the center of the lens, the thickness of the air space is y, and the phase difference for the reflected waves from the path-length difference and the reflection at the bottom surface is

$$\phi = (2y/\lambda)2\pi + \pi.$$

For the bright rings, we have

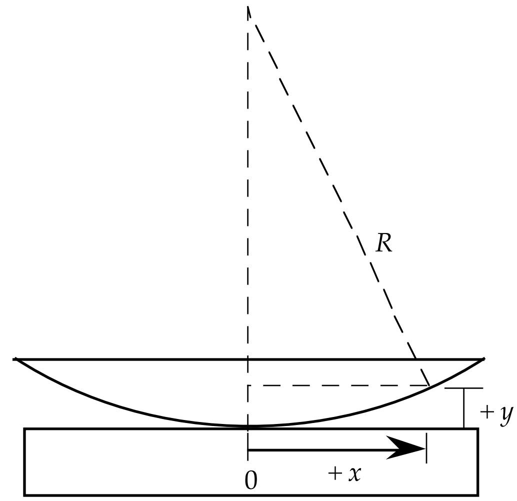
$$\phi = (2y/\lambda)2\pi + \pi = m2\pi, \quad m = 1, 2, 3, \ldots, \quad \text{or}$$
$$y = (m - \tfrac{1}{2})\tfrac{1}{2}\lambda, \quad m = 1, 2, 3, \ldots .$$

For the twentieth bright ring, we have

$$y = (20 - \tfrac{1}{2})\tfrac{1}{2}(560 \text{ nm})$$
$$= 5.46 \times 10^3 \text{ nm} = 5.46\ \mu\text{m}.$$

From the triangle in the diagram, we have

$$x^2 + (R - y)^2 = R^2, \quad \text{or} \quad x^2 = 2yR - y^2 \simeq 2yR, \text{ when } y << R;$$
$$(0.98 \times 10^{-2} \text{ m})^2 = 2(5.46 \times 10^{-6} \text{ m})R, \text{ which gives}$$
$$R = \boxed{8.8 \text{ m}}.$$



43. At a distance x from the center of the lens, the thickness of the air space is y, and the phase difference for the reflected waves from the path-length difference and the reflection at the bottom surface is

$$\phi = (2y/\lambda)2\pi + \pi.$$

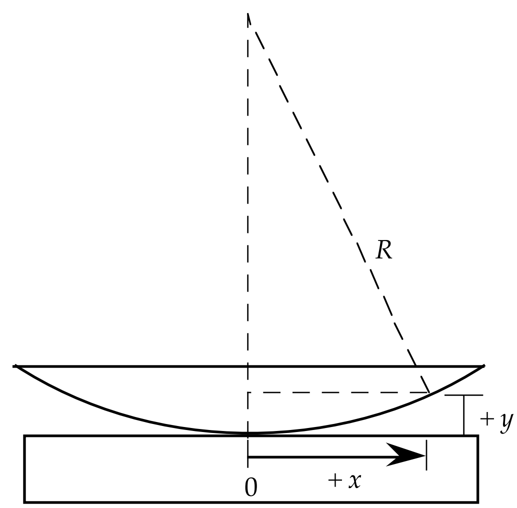
For the dark rings, this phase difference must be an odd multiple of π, so we have

$$\phi = (2y/\lambda)2\pi + \pi = (2n + 1)\pi, \quad n = 0, 1, 2, \ldots, \quad \text{or}$$
$$y = \tfrac{1}{2}n\lambda, \quad n = 0, 1, 2, \ldots .$$

Because $n = 0$ corresponds to the dark center, n represents the number of the ring.
From the triangle in the diagram, we have

$$x^2 + (R - y)^2 = R^2, \quad \text{or} \quad x^2 = 2yR - y^2 \simeq 2yR,$$

when $y << R$.
The position of the nth dark ring is $x = \boxed{(n\lambda R)^{1/2}}$.



44. With respect to the incident wave, the wave that reflects from the soap bubble at the top surface has a phase change of

$$\phi_1 = \pi.$$

With respect to the incident wave, the wave that reflects from the air at the bottom surface of the bubble has a phase change due to the additional path-length but no phase change on reflection:

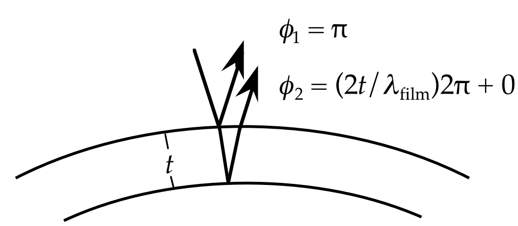
$$\phi_2 = (2t/\lambda_{\text{film}})2\pi + 0.$$

For constructive interference, the net phase change is

$$\phi = (2t/\lambda_{\text{film}})2\pi - \pi = m2\pi, \ m = 0, 1, 2, \ldots, \quad \text{or}$$
$$t = \tfrac{1}{2}\lambda_{\text{film}}(m + \tfrac{1}{2}), \ m = 0, 1, 2, \ldots .$$

The minimum thickness occurs for $m = 0$:

$$t_{\text{min}} = \tfrac{1}{2}(\lambda/n_{\text{film}})(0 + \tfrac{1}{2}) = \tfrac{1}{2}[(460 \text{ nm})/1.35]\tfrac{1}{2} = \boxed{85.2 \text{ nm}}.$$



45. With respect to the incident wave, the wave that reflects from the top surface of the coating has a phase change of

$\phi_1 = \pi.$

With respect to the incident wave, the wave that reflects from the glass ($n \simeq 1.5$) at the bottom surface of the coating has a phase change due to the additional path-length and a phase change of π on reflection:

$\phi_2 = (2t/\lambda_{\text{film}})2\pi + \pi.$

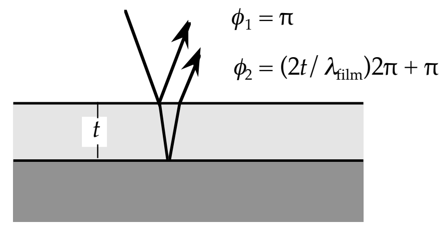
For destructive interference, the net phase change is

$\phi = (2t/\lambda_{\text{film}})2\pi + \pi - \pi = (2m-1)\pi,\ m = 1, 2, 3, \ldots,$ or

$t = \frac{1}{2}\lambda_{\text{film}}(m - \frac{1}{2}),\ m = 1, 2, 3, \ldots .$

The minimum thickness occurs for $m = 1$:

$t_{\min} = \lambda_{\text{film}}/4 = \lambda/4n_{\text{film}} = (650\text{ nm})/4(1.38) = \boxed{118\text{ nm}}.$



46. With respect to the incident wave, the wave that reflects from the top surface of the coating has a phase change of

$\phi_1 = \pi.$

With respect to the incident wave, the wave that reflects from the glass ($n \simeq 1.5$) at the bottom surface of the coating has a phase change due to the additional path-length and a phase change of π on reflection:

$\phi_2 = (2t/\lambda_{\text{film}})2\pi + \pi.$

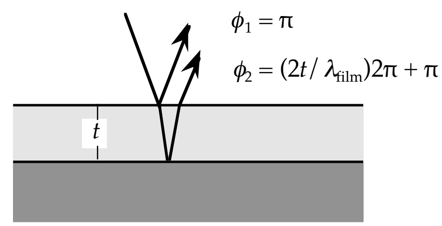
For constructive interference and a nonzero thickness, the net phase change is

$\phi = (2t/\lambda_{\text{film}})2\pi + \pi - \pi = m2\pi,\ m = 1, 2, 3, \ldots,$ or

$t = \frac{1}{2}\lambda_{\text{film}}m,\ m = 1, 2, 3, \ldots .$

The minimum thickness occurs for $m = 1$:

$t_{\min} = \frac{1}{2}\lambda_{\text{film}} = \frac{1}{2}\lambda/n_{\text{film}} = \frac{1}{2}(633\text{ nm})(1.38) = \boxed{229\text{ nm}}.$



47. With respect to the incident wave, the wave that reflects from the oil at the top surface has a phase change of

$\phi_1 = \pi.$

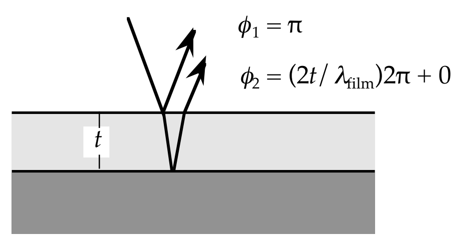
With respect to the incident wave, the wave that reflects from the water at the bottom surface of the oil has a phase change due to the additional path-length but no phase change on reflection:

$\phi_2 = (2t/\lambda_{\text{film}})2\pi + 0.$

For constructive interference, the net phase change is

$\phi = (2t/\lambda_{\text{film}})2\pi - \pi = m2\pi,\ m = 0, 1, 2, \ldots,$ or

$t = \frac{1}{2}\lambda_{\text{film}}(m + \frac{1}{2}) = \frac{1}{2}(\lambda/n_{\text{film}})(m + \frac{1}{2}),\ m = 0, 1, 2, \ldots .$



For the two wavelengths, we have

$t = \frac{1}{2}(\lambda_1/n_{\text{film}})(m_1 + \frac{1}{2}) = \frac{1}{2}(\lambda_2/n_{\text{film}})(m_2 + \frac{1}{2}),$ which gives

$(2m_1 + 1)/(2m_2 + 1) = \lambda_2/\lambda_1 = (682\text{ nm})/(434\text{ nm}) = 1.571.$

By trying the various integers, with $m_1 > m_2$, we find that the set of smallest integers that satisfies the equation is $m_1 = 5$ and $m_2 = 3$.

We find the minimum thickness from

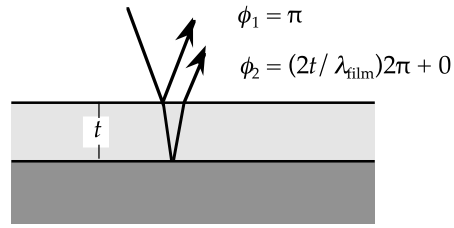
$t = \frac{1}{2}(\lambda_1/n_{\text{film}})(m_1 + \frac{1}{2}) = \frac{1}{2}[(434\text{ nm})/1.51](5 + \frac{1}{2}) = \boxed{790\text{ nm}}.$

48. With respect to the incident wave, the wave that reflects from the oil at the top surface has a phase change of

$\phi_1 = \pi$.

With respect to the incident wave, the wave that reflects from the water at the bottom surface of the oil has a phase change due to the additional path-length but no phase change on reflection:

$\phi_2 = (2t/\lambda_{\text{film}})2\pi + 0$.



For destructive interference and a nonzero thickness, the net phase change is

$\phi = (2t/\lambda_{\text{film}})2\pi - \pi = (2m-1)\pi$, $m = 1, 2, 3, \ldots,$ or

$t = \frac{1}{2}\lambda_{\text{film}}m = \frac{1}{2}(\lambda/n_{\text{film}})m$, $m = 1, 2, 3, \ldots$.

For the three wavelengths, we have

$t = \frac{1}{2}(\lambda_1/n_{\text{film}})m_1 = \frac{1}{2}(\lambda_2/n_{\text{film}})m_2 = \frac{1}{2}(\lambda_3/n_{\text{film}})m_3$, which gives

$m_1/m_2 = \lambda_2/\lambda_1 = (610 \text{ nm})/(550 \text{ nm}) = 1.11 = 20/18$,

$m_1/m_3 = \lambda_3/\lambda_1 = (685 \text{ nm})/(550 \text{ nm}) = 1.245 = 20/16$, and

$m_2/m_3 = \lambda_3/\lambda_2 = (685 \text{ nm})/(610 \text{ nm}) = 1.123 = 18/16$.

We see that the equations are satisfied by $m_1 = 20$, $m_2 = 18$, $m_3 = 16$.

We find the minimum thickness from

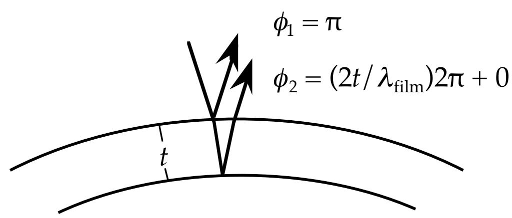
$t = \frac{1}{2}(\lambda_1/n_{\text{film}})m_1 = \frac{1}{2}[(550 \text{ nm})/1.40](20) = 3.93 \times 10^3 \text{ nm} = \boxed{3.93\ \mu\text{m}}$.

49. With respect to the incident wave, the wave that reflects from the top surface of the soap bubble has a phase change of

$\phi_1 = \pi$.

With respect to the incident wave, the wave that reflects from the air at the bottom surface of the bubble has a phase change due to the additional path-length but no phase change on reflection:

$\phi_2 = (2t/\lambda_{\text{film}})2\pi + 0$.



For constructive interference, the net phase change is

$\phi = (2t/\lambda_{\text{film}})2\pi - \pi = m_c 2\pi$, $m_c = 0, 1, 2, \ldots,$ or

$t = \frac{1}{2}\lambda_{\text{film}}(m_c + \frac{1}{2}) = \frac{1}{2}(\lambda_1/n_{\text{film}})(m_c + \frac{1}{2})$, $m_c = 0, 1, 2, \ldots$.

For destructive interference, the net phase change is

$\phi = (2t/\lambda_{\text{film}})2\pi - \pi = (2m_d - 1)\pi$, $m_d = 1, 2, 3, \ldots$, or

$t = \frac{1}{2}\lambda_{\text{film}}m_d = \frac{1}{2}(\lambda_2/n_{\text{film}})m_d$, $m_d = 1, 2, 3, \ldots$.

Because the maximum and minimum are adjacent, $m_d = m_c + 1$, so we have

$t = \frac{1}{2}(\lambda_1/n_{\text{film}})(m_c + \frac{1}{2}) = \frac{1}{2}(\lambda_2/n_{\text{film}})(m_c + 1)$;

$(666 \text{ nm})(m_c + \frac{1}{2}) = (555 \text{ nm})(m_c + 1)$, which gives

$m_c = 2$, and thus $m_d = 3$.

We find the thickness from

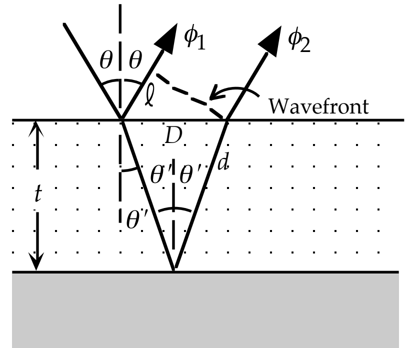
$t = \frac{1}{2}(\lambda_2/n_{\text{film}})m_d = \frac{1}{2}[(555 \text{ nm})/1.34](3) = \boxed{621 \text{ nm}}$.

50. We find the angle of refraction from

$\sin\theta = n_{\text{coat}} \sin\theta'$;

$\sin 45° = 1.41 \sin\theta'$, which gives $\theta' = 30°$.

We must find the phase difference for the two rays along a common wavefront. We do this by referring each ray to the incident point, where the two rays separate. The wave that reflects from the coating has a phase change of



$\phi_1 = (\ell/\lambda)2\pi + \pi$.

The wave that reflects from the glass has a phase change of

$\phi_2 = (2d/\lambda_{\text{coat}})2\pi + \pi = (2dn_{\text{coat}}/\lambda)2\pi + \pi$.

For constructive interference, the net phase change is

$\phi = (2dn_{\text{coat}}/\lambda)2\pi + \pi - [(\ell/\lambda)2\pi + \pi] = m2\pi,\ \ m = 1, 2, 3, \ldots,$ or

$2dn_{\text{coat}} - \ell = m\lambda,\ \ m = 1, 2, 3, \ldots$.

From the figure, we see that

$D = 2t \tan\theta';\quad \ell = D \sin\theta = 2t \tan\theta' \sin\theta;\quad d = t/\cos\theta'$.

When we substitute these in the above equation, we have

$(2n_{\text{coat}}t/\cos\theta') - (2t \tan\theta' \sin\theta) = m\lambda$, or

$t = (m\lambda \cos\theta')/2(n_{\text{coat}} - \sin\theta' \sin\theta) = (m\lambda \cos 30°)/2(1.41 - \sin 30° \sin 45°) = 0.410m\lambda,\ \ m = 1, 2, 3, \ldots$.

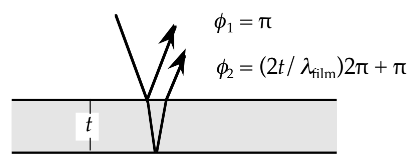
For the red light we have

$t = 0.410(660\text{ nm})m = \boxed{262\text{ nm},\ 525\text{ nm},\ 787\text{ nm}, \ldots}$.

For the blue light we have

$t = 0.410(480\text{ nm})m = \boxed{197\text{ nm},\ 394\text{ nm},\ 590\text{ nm}, \ldots}$.

51. (a) With respect to the incident wave, the wave that reflects from the top surface of the oil film has a phase change of



$\phi_1 = \pi$.

With respect to the incident wave, the wave that reflects from the air at the bottom surface of the bubble has a phase change due to the additional path-length but no phase change on reflection:

$\phi_2 = (2t/\lambda_{\text{film}})2\pi + 0$.

For constructive interference, the net phase change is

$\phi = (2t/\lambda_{\text{film}})2\pi - \pi = m_c 2\pi,\ \ m_c = 0, 1, 2, \ldots,$ or

$t = \frac{1}{2}\lambda_{\text{film}}(m_c + \frac{1}{2}) = \frac{1}{2}(\lambda_1/n_{\text{film}})(m_c + \frac{1}{2}),\ \ m_c = 0, 1, 2, \ldots$.

Because the minimum thickness occurs for $m = 0$, we have

$t_{\min} = \frac{1}{2}[(550\text{ nm})/1.2](0 + \frac{1}{2}) = \boxed{115\text{ nm}}$.

(b) To maintain maximally reflected light, if n were increased, the wavelength would have to be $\boxed{\text{longer}}$, so the ratio remained constant.

(c) Because the index of the film is less than the index of water, there would be an additional phase change of π introduced from the reflection at the bottom surface. What was maximally reflected would now be minimally reflected.

There would be $\boxed{\text{no reflection of green light}}$, but some reflection of red and blue, giving a purple hue.

52. Because the intensity of the light is proportional to the square of the amplitude, we have

$I_r/I_0 = (E_r/E_0)^2 = (n_2 - n_1)^2/(n_2 + n_1)^2$, or $E_r/E_0 = (n_2 - n_1)/(n_2 + n_1)$.

The simplification is that the fraction of the light that reflects from each of the surfaces must be the same to maximize the destructive interference. For the two reflections, we have

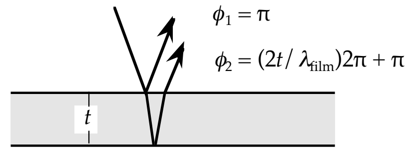
$(E_r/E_0)_{\text{top}} = (E_r/E_0)_{\text{bottom}}$;

$(n_{\text{coat}} - n_{\text{air}})/(n_{\text{coat}} + n_{\text{air}}) = (n_{\text{glass}} - n_{\text{coat}})/(n_{\text{glass}} + n_{\text{coat}})$;

$(1 - n_{\text{air}}/n_{\text{coat}})/(1 + n_{\text{air}}/n_{\text{coat}}) = (1 - n_{\text{coat}}/n_{\text{glass}})/(1 + n_{\text{coat}}/n_{\text{glass}})$;

$n_{\text{air}}/n_{\text{coat}} = n_{\text{coat}}/n_{\text{glass}}$.

53. With respect to the incident wave, the wave that reflects from the top surface of the coating (n = 1.25) has a phase change of



$$\phi_1 = \pi.$$

With respect to the incident wave, the wave that reflects from the glass (n = 1.55) at the bottom surface has a phase change due to the additional path-length and reflection:

$$\phi_2 = (2t/\lambda_{film})2\pi + \pi.$$

For destructive interference, the net phase change is

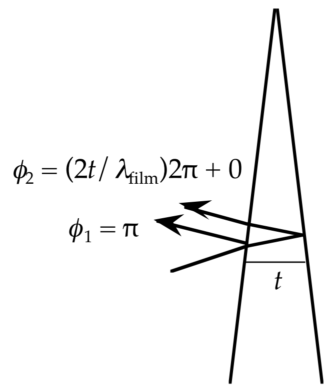
$$\phi = (2t/\lambda_{film})2\pi + \pi - \pi = (2m - 1)\pi,\ \ m = 1, 2, 3, \ldots,\ \text{ or}$$

$$t = \tfrac{1}{2}\lambda_{film}(m - \tfrac{1}{2}),\ \ m = 1, 2, 3, \ldots .$$

The minimum thickness occurs for $m = 1$:

$$t_{min} = \lambda_{film}/4 = \lambda/4n_{film} = (600\text{ nm})/4(1.25) = \boxed{120\text{ nm}}.$$

54. With respect to the incident wave, the wave that reflects from the soap film at the front surface has a phase change of



$$\phi_1 = \pi.$$

With respect to the incident wave, the wave that reflects from the air at the back surface of the film has a phase change due to the additional path-length but no phase change on reflection:

$$\phi_2 = (2t/\lambda_{film})2\pi + 0.$$

For constructive interference, the net phase change is

$$\phi = (2t/\lambda_{film})2\pi - \pi = m2\pi,\ \ m = 0, 1, 2, \ldots,\ \text{ or}$$

$$t = \tfrac{1}{2}\lambda_{film}(m + \tfrac{1}{2}) = \tfrac{1}{2}(\lambda/n_{film})(m + \tfrac{1}{2}),\ \ m = 0, 1, 2, \ldots .$$

The change in thickness between adjacent bright bands is

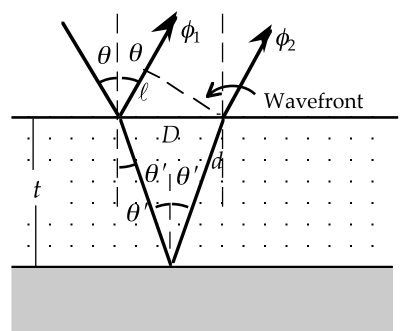
$$\Delta t = \tfrac{1}{2}(\lambda/n_{film})\ \Delta m = \tfrac{1}{2}(\lambda/n_{film}).$$

The rate of change of thickness with height is

$$\Delta t/\Delta h = \tfrac{1}{2}(\lambda/n_{film})/\Delta h = \tfrac{1}{2}(\lambda/n_{film}\ \Delta h)$$

$$= \tfrac{1}{2}(485 \times 10^{-9}\text{ m})/(1.36)(0.6 \times 10^{-2}\text{ m}) = \boxed{30\ \mu\text{m/m}}.$$

55. We find the angle of refraction from



$$\sin\theta = n_{coat}\sin\theta';$$

$$\sin 30° = 1.25\sin\theta',\ \text{which gives } \theta' = 23.6°.$$

We must find the phase difference for the two rays along a common wavefront, indicated on the figure. We do this by referring each ray to the incident point, where the two rays separate. The wave that reflects from the coating has a phase change of

$$\phi_1 = (\ell/\lambda)2\pi + \pi.$$

The wave that reflects from the glass has a phase change of

$$\phi_2 = (2d/\lambda_{coat})2\pi + \pi = (2dn_{coat}/\lambda)2\pi + \pi.$$

For destructive interference, the net phase change is

$$\phi = (2dn_{coat}/\lambda)2\pi + \pi - [(\ell/\lambda)2\pi + \pi] = (2m - 1)\pi,\ m = 1, 2, 3, \ldots .$$

For the minimum thickness, $m = 1$, we have

$$(2dn_{coat}/\lambda) - (\ell/\lambda) = \tfrac{1}{2}.$$

From the figure, we see that

$$D = 2t\tan\theta';\ \ \ell = D\sin\theta = 2t\tan\theta'\sin\theta;\ \ d = t/\cos\theta'.$$

When we substitute these in the above equation, we have

$$(2t/\lambda)[n/(\cos\theta') - \tan\theta'\sin\theta] = \tfrac{1}{2};$$

$$[2t/(550\text{ nm})][1.25/(\cos 23.6°) - \tan 23.6°\sin 30°] = \tfrac{1}{2},\ \text{which gives } t = \boxed{120\text{ nm}}.$$

56. We find the number of fringes from

$N = 2\,\Delta L/\lambda = 2(0.10 \times 10^{-3}\text{ m})/(633 \times 10^{-9}\text{ m}) = \boxed{316\text{ fringes}}$.

57. We find the wavelength from

$N = 2\,\Delta L/\lambda$;

$980 = 2(0.31 \times 10^{-3}\text{ m})/\lambda$, which gives $\lambda = 6.33 \times 10^{-7}\text{ m} = \boxed{633\text{ nm}}$.

58. The phase shift produced by the introduction of the glass is produced by the change in the number of wavelengths in the distance equal to the thickness of the glass:

$\phi = (2t/\lambda_{\text{glass}} - 2t/\lambda)2\pi = (2tn_{\text{glass}}/\lambda - 2t/\lambda)2\pi = (2t/\lambda)(n_{\text{glass}} - 1)2\pi$.

Because each 2π phase shift corresponds to a fringe shift, we have

$(2t/\lambda)(n_{\text{glass}} - 1) = N$;

$[2t/(426 \times 10^{-9}\text{ m})](1.55 - 1) = 750$, which gives $t = 2.90 \times 10^{-4}\text{ m} = \boxed{0.290\text{ mm}}$.

59. The phase shift produced by the introduction of the gas that fills one of the glass tubes of length t is

$\phi = (2t/\lambda_{\text{gas}} - 2t/\lambda)2\pi = (2tn_{\text{air}}/\lambda - 2t/\lambda)2\pi = (2t/\lambda)(n_{\text{gas}} - 1)2\pi$.

Because each 2π phase shift corresponds to a fringe shift, we have

$(2t/\lambda)(n_{\text{gas}} - 1) = N$.

The wavelength of the light source is not given in the problem statement. If we assume the source to be the common He-Ne laser, then $\lambda = 632.8$ nm; so

$[2(0.10\text{ m})/(632.8 \times 10^{-9}\text{ m})](n_{\text{gas}} - 1) = 90$, which gives $n = \boxed{1.00028}$.

60. This problem is similar to Problem 58. The phase shift produced by the introduction of the glass plate is produced by the change in the number of wavelengths in the distance equal to the thickness of the glass plate:

$\phi = (2t/\lambda_{\text{glass}} - 2t/\lambda)2\pi = (2tn_{\text{glass}}/\lambda - 2t/\lambda)2\pi = (2t/\lambda)(n_{\text{glass}} - 1)2\pi$.

Because each 2π phase shift corresponds to a fringe shift, we have

$(2t/\lambda)(n_{\text{glass}} - 1) = N$;

$[2t/(589 \times 10^{-9}\text{ m})](1.58 - 1) = 25$, which gives $t = 1.3 \times 10^{-5}\text{ m} = \boxed{0.013\text{ mm}}$.

61. The number of fringe shifts produced by a mirror movement of ΔL is

$N = 2\,\Delta L/\lambda$.

For the minimum mirror movement, the minimum number of fringes is

$N_{\min} = 2(0.03 \times 10^{-3}\text{ m})/(590 \times 10^{-9}\text{ m}) = 102$.

The uncertainty in the number due to the uncertainty in the wavelength is

$dN = -(2\,\Delta L/\lambda^2)\,d\lambda$, or $dN/N = -d\lambda/\lambda = 0.1\%$.

$\boxed{\text{A minimum of approximately 100 fringes must be counted to 0.1 of a fringe}}$.

62. For the two wavelengths, we have

$N_1 = 2\,\Delta L/\lambda_1$ and $N_2 = 2\,\Delta L/\lambda_2$.

When we divide these two equations, we have

$N_1/N_2 = \lambda_2/\lambda_1$;

$714/593 = \lambda_2/(582.5\text{ nm})$, which gives

$\lambda_2 = \boxed{701.4\text{ nm}}$.

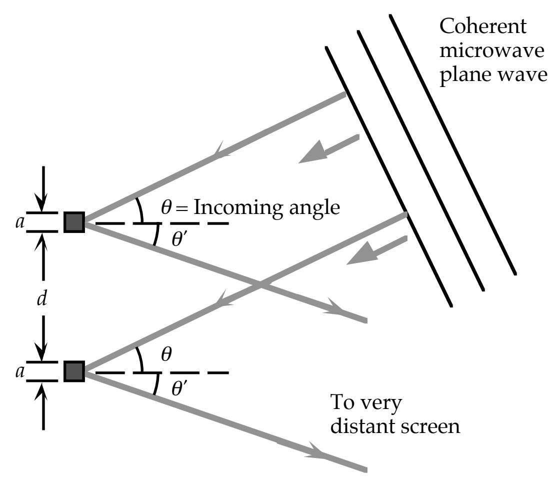
$\boxed{63}$. Because the screen is far from both light bulbs, their intensities will be the same. Because the sources are not coherent, there will be no interference. The intensity will be $\boxed{\text{uniform}}$.

64. If L is the distance from the transmitting tower to your home and D is the distance of the reflecting tower off to the side, the path-length difference causing the interference is
$\Delta L = 2[(\frac{1}{2}L)^2 + D^2]^{1/2} - L.$
Because $D << L$, this becomes
$\Delta L = L[1 + (2D/L)^2]^{1/2} - L \simeq L[1 + 2(D/L)^2] - L = 2D^2/L.$
To produce destructive interference, this path-length difference must be a half-wavelength:
$\frac{1}{2}\lambda = 2D^2/L;$
$\frac{1}{2}(480 \text{ m}) = 2D^2/(13 \times 10^3 \text{ m})$, which gives $D = 1.25 \times 10^3 \text{ m} = \boxed{1.25 \text{ km}}$.

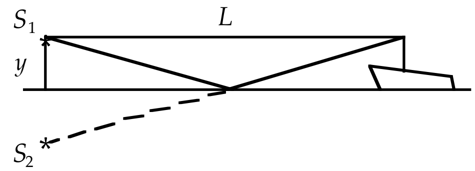
65. We neglect the variation in amplitude:
$E_1 \simeq E_2 = E.$
Because the sources radiate in phase, they will be in phase at the midpoint, $x' = 0$.
Between the sources the waves travel in opposite directions, so we have
$$\begin{aligned} E_{\text{net}} &= E \sin(kx' - \omega t) + E \sin(kx' + \omega t) \\ &= E[\sin(kx' - \omega t) + \sin(kx' + \omega t)] \\ &= 2E \sin(kx') \cos(\omega t) \\ &= 2E \sin(2\pi f x'/c) \cos(2\pi f t) \end{aligned}$$
We find the energy density from
$u = \langle \frac{1}{2}\varepsilon_0 E_{\text{net}}^2 \rangle = 2\varepsilon_0 E^2 \sin^2(2\pi f x'/c) \langle \cos^2(2\pi f t) \rangle.$
The time average of $\langle \cos^2 \theta \rangle = \frac{1}{2}$. If we change to the distance from one source, $x = x' + \frac{1}{2}L$, we have
$\boxed{u \propto \sin^2[2\pi f(x - \frac{1}{2}L)/c]}$.

66. We neglect the variation in amplitude:
$E_1 \simeq E_2 = E.$
We find the phase difference produced by the additional path-length:
$\phi = (2L/\lambda)2\pi = (2Lf/c)2\pi = [2(100 \text{ m})(97.9 \times 10^6 \text{ Hz})/(3 \times 10^8 \text{ m/s})]2\pi = 4.10 \times 10^2 \text{ rad}.$
We take $x = 0$ at the radio. Because the waves are traveling in opposite directions, we have
$E_{\text{net}} = E \sin(-\omega t) + E \sin(+\omega t + \phi) = 2E \sin(\frac{1}{2}\phi) \cos(\omega t).$
Because the intensity is proportional to the time average of the electric field, we have
$I/I_0 = \langle E_{\text{net}}^2 \rangle / \langle E_1^2 \rangle = 4E^2 \sin^2(\frac{1}{2}\phi) \langle \cos^2(\omega t) \rangle / E^2 \langle \sin^2(-\omega t) \rangle.$
Because $\langle \cos^2(\omega t) \rangle = \langle \sin^2(-\omega t) \rangle = \frac{1}{2}$, we have
$I/I_0 = 4 \sin^2[\frac{1}{2}(4.10 \times 10^2 \text{ rad})] = \boxed{2.04}$.

67. Because the obstacles are much smaller than the wavelength, the reflections radiate uniformly in all directions. The maxima will be determined by the phase shift only. When the incoming wavefront reaches the top obstacle, it still has to travel a distance $L_1 = d \sin \theta$ to reach the bottom obstacle. When it reaches the bottom obstacle, the wave reflecting from the top obstacle will have traveled a distance $L_2 = d \sin \theta'$.
For maxima, the net path-length difference must be a multiple of the wavelength:
$$\begin{aligned} L_1 - L_2 &= (d \sin \theta) - (d \sin \theta') \\ &= m\lambda, \ m = 0, \pm 1, \pm 2, \ldots, \text{ which gives} \end{aligned}$$
$\boxed{\sin \theta' = \sin \theta - (m\lambda/d), \ m = 0, \pm 1, \pm 2, \ldots}$.



68. The reflected radio wave appears to be coming from the image of the transmitter, so we can treat this as a double-slit
problem; however, there is an additional phase shift of π because of the reflection.



For destructive interference, the net phase shift is

$$\phi = [d(\sin\theta)/\lambda]2\pi + \pi = (2m + 1)\pi, \quad m = 0, 1, 2, \ldots,$$

where $d = 2y$ and $m = 0$ corresponds to a receiver far away.
If we assume small angles, $\sin\theta \simeq \tan\theta = y/L$, we have

$$(2y)y/L\lambda = 2y^2f/Lc = m, \quad m = 1, 2, \ldots .$$

We find the distance for the first minimum, $m = 1$, from

$$2(15 \text{ m})^2(230 \times 10^6 \text{ Hz})/L_1(3 \times 10^8 \text{ m/s}) = 1, \text{ which gives } L_1 = 345 \text{ m}.$$

The distance at which the second minimum, $m = 2$, occurs is

$$L_2 = L_1/m = (345 \text{ m})/2 = 173 \text{ m}.$$

We find the speed of the ship from

$$v = \Delta L/\Delta t = (345 \text{ m} - 173 \text{ m})/(170 \text{ s}) = \boxed{1.01 \text{ m/s}}.$$

69. From Problem 26, we have

$$\langle I_{net}\rangle = \langle I_1\rangle + \langle I_2\rangle + 2(\langle I_1\rangle\langle I_2\rangle)^{1/2}\cos\phi.$$

When $I_1 = 2I_2$, this becomes

$$\langle I_{net}\rangle = 2\langle I_2\rangle + \langle I_2\rangle + 2(\langle 2I_2\rangle\langle I_2\rangle)^{1/2}\cos\phi$$
$$= \langle I_2\rangle(3 + 2\sqrt{2}\cos\phi).$$

The maximum intensity will occur when $\cos\phi = 1$, and the minimum intensity will occur when $\cos\phi = -1$. For the ratio, we have

$$\langle I_{max}\rangle/\langle I_{min}\rangle = (3 + 2\sqrt{2})/(3 - 2\sqrt{2}) = \boxed{34.0}.$$

70. (a) When the signals are in phase, we find the phase difference from the path-length difference:

$$\phi = (\Delta L/\lambda)2\pi = \{[(\lambda/2)\cos\theta]/\lambda\}2\pi = \pi\cos\theta.$$

The radiated intensity is

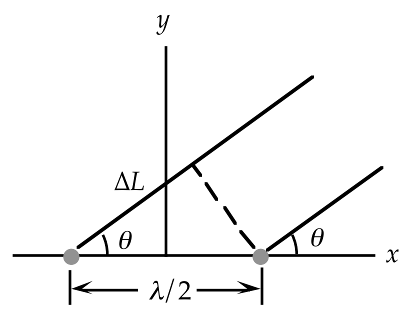
$$I = 4I_0\cos^2(\phi/2) = \boxed{4I_0\cos^2[(\pi\cos\theta)/2]}.$$

The intensity will be

$$\boxed{\text{minimum when } \cos\theta = \pm 1, \theta = 0° \text{ and } 180°}.$$

The intensity will be

$$\boxed{\text{maximum when } \cos\theta = 0, \theta = 90° \text{ and } 270°}.$$



(b) When the signals are 180° out of phase, there will be an additional phase shift of π:

$$\phi = (\Delta L/\lambda)2\pi + \pi = \{[(\lambda/2)\cos\theta]/\lambda\}2\pi + \pi = \pi(1 + \cos\theta).$$

The radiated intensity is

$$I = 4I_0\cos^2(\phi/2) = \boxed{4I_0\cos^2[\pi(1 + \cos\theta)/2]}.$$

The intensity will be $\boxed{\text{minimum when } \cos\theta = 0, \ \theta = 90° \text{ and } 270°}$.

The intensity will be $\boxed{\text{maximum when } \cos\theta = \pm 1, \ \theta = 0° \text{ and } 180°}$.

71. When the signals are 90° out of phase, the total phase difference is

$$\phi = (\Delta L/\lambda)2\pi + \pi/2 = \{[(\lambda/4)\cos\theta]/\lambda\}2\pi + \pi/2 = (\pi/2)(1 + \cos\theta).$$

The radiated intensity is

$$I = 4I_0\cos^2(\phi/2) = 4I_0\cos^2[(\pi/4)(1 + \cos\theta)].$$

The intensity will be maximum when $(\pi/4)(1 + \cos\theta) = 0$ or π, which gives

$$1 + \cos\theta = 0 \text{ or } 4; \quad \cos\theta = -1 \text{ or } 3.$$

Because $\cos\theta = 3$ is impossible, there is only one maximum at $\theta = 180°$.

72. At the maxima all three waves must be in phase. Thus between two adjacent slits, we must have the same condition as for a double slit: $\Delta L = n\lambda$.
From Problem 10 we have

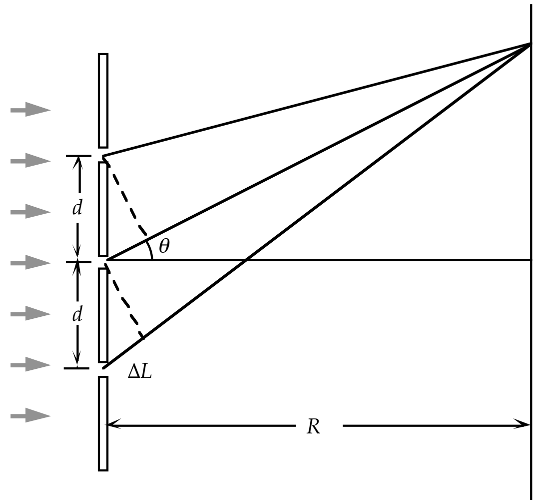
$$y_{\max} = \boxed{(n - \tfrac{1}{2})R\lambda/d, \quad n = 0, \pm 1, \pm 2, \dots} .$$

At the maxima the total electric field amplitude is

$$E = 3E_0.$$

The intensity depends on the (amplitude)^2, so we have

$$I = \boxed{9I_0}.$$



73. For a listener along the diagonal, $\theta = 45°$. Because the signals are in phase, we find the phase difference for the signals from A and B from the path-length difference:

$$\phi_{AB} = (\Delta L/\lambda)2\pi = \{[(\lambda/\sqrt{2})\cos 45°]/\lambda\}2\pi = \pi.$$

The radiated intensity is

$$I_{AB} = 4I_0 \cos^2(\phi_{AB}/2) = 4I_0 \cos^2(\pi/2) = 0.$$

From the diagram, we see that

$\phi_{AD} = \phi_{BC} = \phi_{CD} = \phi_{AB} = \pi$, which gives

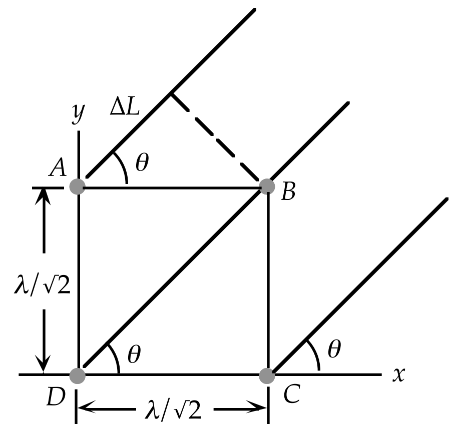
$$I_{AD} = I_{BC} = I_{CD} = I_{AB} = 0.$$

For the pair AC, we have

$\phi_{AC} = 0$, which gives $I_{AC} = 4I_0$.

For the pair BD, we have

$\phi_{BD} = 2\phi_{AB} = 2\pi$, which gives $I_{BD} = 4I_0$.



There are four possible combinations of three speakers:

A, B, and C; A, B, and D; A, C, and D; B, C, and D.

Each of these combinations has a pair out of phase, which leaves the equivalent of one speaker:

$$I_{ABC} = I_{ABD} = I_{ACD} = I_{BCD} = I_0.$$

The combination of four speakers has two pairs, each out of phase:

$$I_{ABCD} = 0.$$

In summary:

AB, AD, BC, CD	$I = \boxed{0}$
AC, BD	$I = \boxed{4I_0}$
ABC, ABD, ACD, BCD	$I = \boxed{I_0}$
$ABCD$	$I = \boxed{0}$.

74. Consider the two light rays, a and b, in the figure to the right. The path-length difference between the two rays is

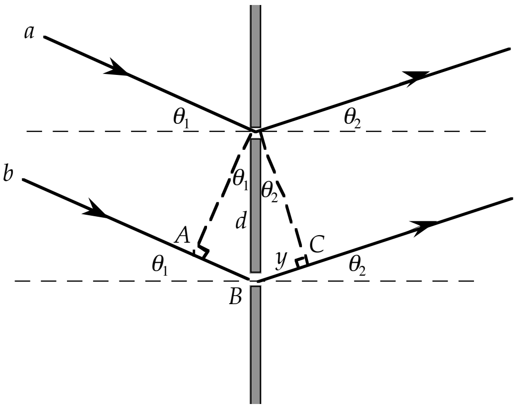
$\Delta x = AB + BC = d \sin \theta_1 + d \sin \theta_2$.

Constructive interference occurs when

$\Delta x = m\lambda$;

$d \sin \theta_1 + d \sin \theta_2 = = m\lambda$; so

$m\lambda / d = \sin \theta_1 + \sin \theta_2$, where m is an integer.

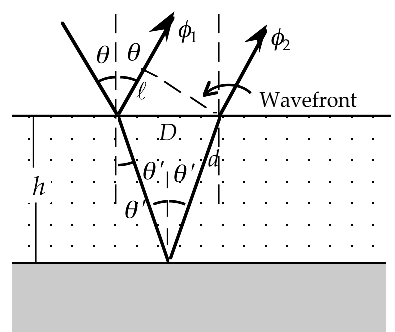


75. We find the angle of refraction from

$\sin \theta = n \sin \theta'$.

We must find the phase difference for the two rays along a common wavefront, indicated on the figure. We do this by referring each ray to the incident point, where the two rays separate. The wave that reflects from the coating has a phase change of

$\phi_1 = (\ell/\lambda)2\pi + \pi$.



The wave that reflects from the mirror has a phase change of

$\phi_2 = (2d/\lambda_{\text{glass}})2\pi + 0 = (2nd/\lambda)2\pi$.

For constructive interference, the net phase change is

$\phi = (2nd/\lambda)2\pi - [(\ell/\lambda)2\pi + \pi] = m2\pi, \ m = 1, 2, 3, \ldots$;

$(2nd/\lambda) - (\ell/\lambda) = m - \frac{1}{2}, \ m = 1, 2, 3, \ldots$.

From the figure, we see that

$D = 2h \tan \theta'$;

$\ell = D \sin \theta = 2h \tan \theta' \sin \theta$;

$d = h/\cos \theta'$.

When we substitute these in the above equation, we have

$(2h/\lambda)\{[n/(\cos \theta')] - (\tan \theta' \sin \theta)\} = m - \frac{1}{2}, \ m = 1, 2, 3, \ldots$.

We reduce the term in { } by using the result from the refraction:

$$n/(\cos \theta') - \tan \theta' \sin \theta = (n - \sin \theta' \sin \theta)/\cos \theta'$$
$$= [n - (\sin^2 \theta)/n]/[1 - (\sin^2 \theta)/n^2]^{1/2}$$
$$= (n^2 - \sin^2 \theta)^{1/2}.$$

The condition for maxima becomes

$(2h/\lambda)(n^2 - \sin^2 \theta)^{1/2} = m - \frac{1}{2}, \ m = 1, 2, 3, \ldots$, which gives

$\sin^2 \theta = n^2 - [\lambda(m - \frac{1}{2})/2h]^2$;

$\theta = \boxed{\sin^{-1} \{n^2 - [\lambda(m - \frac{1}{2})/2h]^2\}^{1/2}, \ m = 1, 2, 3, \ldots}$.

76. (a) As S and I get arbitrarily close, there will be no path-length difference between the ray from the point source and the ray from the image. Because there is a π phase shift for the reflected light from the image, the two rays will interfere destructively.

(b) We can treat the source and image as a double-slit system with an additional π phase shift. If h is the height of the source above the plane of the mirror, for the maxima we have

$\phi = [(2h \sin \theta)/\lambda]2\pi + \pi = m2\pi, \ m = 1, 2, 3, \ldots$, which gives

$\sin \theta = (\lambda/2h)(m - \frac{1}{2}), \ m = 1, 2, 3, \ldots$.

When the screen is far away, the angles will be small, with $\sin \theta \simeq \theta$, so we have

$\theta = (\lambda/2h)(m - \frac{1}{2}), \ m = 1, 2, 3, \ldots$.

The angular separation between maxima is

$\Delta\theta = (\lambda/2d)\,\Delta m = [\lambda/2(0.50 \times 10^{-3}\ \text{m})](1) = \boxed{10^3 \lambda \text{ rad, with } \lambda \text{ in m}}$.